

Portfolio Optimization: A Practical Approach Using Monte Carlo Simulation and Machine Learning Method

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Abstract

This paper explores the application of Harry Markowitz's Modern Portfolio Theory (MPT) through two distinct computational lenses: stochastic Monte Carlo simulations and deterministic mathematical optimization. Using a portfolio comprising major technology and hardware equities: Apple (AAPL), Cisco (CSCO), IBM, and Amazon (AMZN). This study evaluates the risk-return trade-offs during a specific historical period. We analyse the construction of the Efficient Frontier and identify the Optimal Risky Portfolio based on the maximization of the Sharpe Ratio.

Keywords

Efficient Frontier, Log Returns, Modern Portfolio Theory, Monte Carlo Simulation, Portfolio Optimization, Sharpe Ratio.

INTRODUCTION

Portfolio optimisation is impactful functions in the theory of quantitative finance. The idea behind this is that most investors are risk-averse, thus they will prefer the portfolio with extremely high returns but at a minimum level of risk. In this computation structure, we are applying the basic idea of Modern Portfolio Theory (MPT). Here, the set of historical daily closing prices of four assets has been used to find the most efficient set of weight which increases the risk adjusted return of the portfolio. Efficient Portfolio Theory (E.P.T), developed by [1] has identified that investors can increase return and reduce risk through diversification. Modern Portfolio Theory does not view assets individually but instead focuses on how the returns from various securities covary and lists the risk of portfolio as a function of the covariances among the asset returns. It has laid the foundations for modern investment strategy and illustrates the trade between return and risk. By enabling the quantitative construction of diversified portfolios aligned with the statistical properties of quantified risks and returns, the MPT has been revolutionary in both portfolio management and financial economics. Nevertheless, the MPT theory makes several assumptions, such as using historical data and assuming returns to be normally distributed, correlations to be constant, and that there are no exogenous factors other than financial data. There have been several models and extensions to address these issues.

Two foundational financial theories to provide context for portfolio optimization:

1. **Modern Portfolio Theory (MPT):** This illustrates the trade-off theory between return and risk, focusing on how the returns from various securities covary rather

than viewing assets individually.

2. **Efficient Portfolio Theory (E.P.T):** In 1952, Harry Markowitz developed this theory established that investors can increase their returns and reduce risk through diversification.

This paper discusses about how portfolio optimisation can be derived through quantitative methods supported by Python computations. Quantitative steps follow, starting by a systematic portfolio data pre-processing, log-returns calculation (time-additivity adjustment and statistical normalization), a Monte Carlo simulation procedure to generate an arbitrary large sample of randomly weighted portfolios for an empirical risk-return plot, applying established constrained numerical optimization procedures to calculate the minimum variance portfolio and the portfolio with maximum Sharpe ratio, constructing and plotting out the Empirical Efficient Frontier. By using financial theory alongside data analysis tools, this method helps create practical ways to choose a portfolio that fits real-world data.

LITERATURE REVIEW

Traditional mean-variance portfolio optimization introduced by [1] has long been regarded as one of the fundamental theories of portfolio management. However, its application in real financial markets involves several practical difficulties. One of the major challenges is that the model relies heavily on estimates of expected returns and asset covariance, which are generally calculated using historical data. When the available data is limited, these estimates may become inaccurate and lead to unreliable investment decisions. In addition, the model assumes that returns are normally distributed and that relationships among assets remain stable over time, assumptions that are often

unrealistic in dynamic financial markets. As noted by [2], these limitations may result in unstable portfolio weights and weaker performance under actual market conditions.

Because of these limitations, researchers have increasingly adopted machine learning techniques for return forecasting and portfolio optimization. Many recent studies combine machine learning approaches with simulation methods such as Monte Carlo analysis to improve forecasting accuracy and investment decision-making. For example, [3] discussed the application of advanced approaches, including machine learning and quantum-inspired algorithms, for supporting investment decisions in highly volatile and uncertain market environments. Similarly, [4] examined the use of clustering techniques, support vector machines, and genetic algorithms to identify complex relationships among financial assets, while also highlighting issues such as model overfitting.

Recent studies also treat portfolio construction as a two-stage process involving parameter estimation followed by optimization. In this context, [5] explained that machine learning models are capable of adapting to changing market conditions by using large and diverse datasets, including alternative financial information and nonlinear relationships among assets. In a similar direction, [6] applied clustering, regression, and classification methods to estimate covariance structures and further explored reinforcement learning and deep learning techniques for portfolio rebalancing and asset dependency analysis.

An important contribution to this field was made by [2], who incorporated machine learning-based regularization and cross-validation techniques directly into the portfolio optimization framework. Their approach aimed to reduce the impact of estimation errors and improve the stability and reliability of portfolio performance in real market situations.

In general, they find PBR to outperform SAA, L_1 and L_2 regularizations, and naive equal weighted portfolios. Monte Carlo simulation offers an additional approach to building a portfolio in modern finance. It can help generate scenarios, determine efficient frontiers, propagate uncertainty, and can also be used to solve stochastic optimization problems. [7] predicts stocks return using a sliding-window linear regression model and applies MC simulation with the estimated variance-covariance matrix. The resulting Maximum Sharpe or Minimum risk portfolio beats CSI 300 Index. More comparative research confirms this trend. [8] compares the effectiveness of Markowitz optimization, MC simulation and LASSO algorithm on NASDAQ stocks, showing the rising popularity of sparse ML techniques in weight estimation problem. In a unified framework, [9] propose a multi-stage strategy where ML techniques predict returns, top assets are selected, weights are optimized through MC simulation and trading signals are issued by classifiers trained with technical indicators. The optimized portfolio outperforms single-step trading strategies. Monte Carlo methods are also useful tools for managing fixed-income portfolios. [10] present a two-stage stochastic bond portfolio optimization problem, in which interest rate

and liabilities are uncertain. They change the stochastic problem to a deterministic mixed integer linear program and show to minimize the cost. More recently, the idea of decision focused learning and end to end learning has gained attention. [11] showed that end-to-end training of a ML model through minimizing losses based on portfolio allocations, rather than decoupling prediction and optimization steps, is more efficient than traditional sequential modelling. [12] reviews several innovations on deep learning, reinforcement learning and ensemble learning techniques for asset return prediction, risk prediction and optimal weights determination. Results shows that ML techniques can increase expected returns, reduce downside risk, and improve diversification compared to traditional optimization. In summary, the optimization literature has evolved from traditional, purely statistical mean-variance optimization to hybrid models combining machine learning algorithms and Monte Carlo simulation. Such approaches enable the comprehensive modelling of market uncertainty, facilitate greater forecasting accuracy and a more robust optimization using a regularization approach, and provide a more reliable out of sample performance. It is therefore implied that portfolio selection when enhanced with ML techniques is a useful step forward from an otherwise traditional approach.

DATA ANALYSIS

This analysis uses daily closing prices for four major stocks: Apple (AAPL), Cisco (CSCO), IBM (IBM), and Amazon (AMZN).

The data were retrieved using financial APIs, like Yahoo Finance [13]. To make the analysis easier, the risk-free rate is supposed to be zero percent, all figures for performance related measures are converted to the yearly equivalent (assuming 252 trading days per year). The data used from 3 January 2022 to 29 December 2023. Table 1 summarizes the relevant descriptive statistical summary of the daily returns for the YTD stocks, documenting the five-point summary of the daily returns during 252 trading days.

Table 1. Descriptive Statistics of Daily Returns (January 2022 – December 2023)

	AAPL	CISCO	IBM	AMZN
Min	-0.058680	-0.137304	-0.052487	-0.140494
Max	0.088974	0.058080	0.071003	0.135359
Mean	0.000302	-0.000196	0.000643	0.000129
Variance	0.000335	0.000253	0.000162	0.000719

Sources: Data computed by the author based on [Financial APIs, Yahoo Finance, 2022-23].

The descriptive statistics allows us to have a glance at the return distribution and volatility features of four selected stock, Amazon (AMZN), Apple (AAPL), Cisco System (CSCO) and International Business Machine (IBM) during the period. IBM was the best of the four, although it had the

lowest variance of all the assets in the universe. It had the highest average daily return, (0.000643), and the lowest variance (0.000162) of all the assets. In portfolio theory, higher average combined with a lower variance is an attractive characteristic, as it exudes risk without much sacrifice to expected return, IBM has this characteristic. Conversely, Amazon appears to be the most volatile stock, with the highest single-day increase (13.5%) and single-day drop (-14.0%) and highest variance (0.000719). Amazon shows a high growth, high-risk profile. While increased volatility will undoubtedly enlarge any given portfolio's risk, the potential for upside should be noted; thus, presenting an excellent candidate for inclusion in a portfolio with a high-risk appetite. The only stock in the sample with a negative average daily return is Cisco (-0.000196). With a variance comparable to those of Amazon and Apple, its negative mean implies it did not perform as well during the sample period. In an optimization problem, a stock with negative mean should always receive a smaller weight unless the correlation with the other assets is very low, in which case the contribution to portfolio efficiency may be positive. The portfolio presents a mixture of both stability and growth, while apple is somewhere into the middle of the spectrum. The mean return is positive, and not too high (0.000302) and the variance is also moderate (0.000335).

Relative to IBM, apple also has higher volatility, but it is much more stable than amazon. Therefore, it can be seen as an asset with growth potential that has flexible risk, acting as a bridge between defensive and aggressive investment assets. In general, the statistical features of these assets underscore the trade between risk and return. These features constitute the empirical basis for portfolio optimizers, where the weights of assets are selected to realize the maximum expected return for given risks and the benefits of diversification [1].

METHODOLOGY

Software and Tools Required

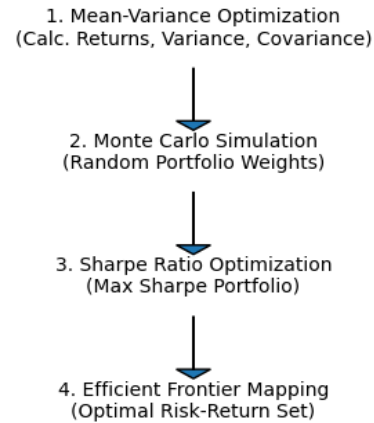
The methodology for portfolio management is structurally supported by several software tools, as outlined in the work of [14]. The computational environment is built on Python, utilizing NumPy for numerical array and matrix operations, Pandas for handling and cleaning time-series stock data, and Matplotlib for creating visual plots of the efficient frontier. Additionally, Scikit-learn provides the necessary machine learning techniques for optimization and predictive modelling, all of which are integrated into an interactive Jupyter Notebook.

METHOD

Figure 1 presents a practical and commonly adopted workflow for portfolio optimization based on the theory developed by [1], i.e, Modern Portfolio Theory. The framework also integrates simulation techniques to address real-world market uncertainty and support the development

of more stable and risk-adjusted investment decisions.

Typical Workflow Sequence in Portfolio Optimization



Sources: Created by author.

Figure 1. Portfolio Optimization workflow

Efficient Frontier

This method used to show all probable combinations of investments on a graph. The vertical axis of frontier graph shows how much money you can make, and the horizontal axis shows how risky each investment is. This idea was created by Harry Markowitz, who won a Nobel Prize in the year 1952. It's a key part of Modern Portfolio Theory, which helps people choose investments that give the best return for the amount of risk they're willing to take. This concept is also called the Markowitz Efficient Set or the Markowitz Model. The Markowitz model uses how much each individual investment is expected to earn and how much it might change in value to figure out the overall expected return and risk of the whole group of investments. The expected return of a portfolio is calculated as follows:

$$E(R_p) = w_1 * E(R_1) + w_2 * E(R_2) + \dots + w_n * E(R_n)$$

$E(R_p)$ expected return of the portfolio, w_i weight of asset i in the portfolio and $E(R_i)$ expected return of asset i .

Investors can use the efficient frontier to choose portfolios that best match the risk tolerance and financial objectives. Through understanding the balance between expected returns and risk investors can make better decisions and create diversified portfolios that align with their individual investment needs [1].

Monte Carlo Simulation

[15], a mathematician and physicist who worked on the Manhattan Project, which aimed to create the first atomic bomb, started developing the Monte Carlo method in 1946 [16]. Nicholas Metropolis and Stanislaw Ulam introduced the method in a landmark 1949 paper published in the Journal of the American Statistical Association: Monte Carlo technique inspired by Stanislaw Ulam's passion for the game of poker, Metropolis offered the name Monte Carlo to refer to the statistical simulation method [17]. Based on probabilistic analysis, Monte Carlo Simulation is a widely used method for simulating the behaviour of complex systems and predicting

outcomes. To find the optimal portfolio with the highest return and lowest risk, this method can be used to simulate different scenarios and generate random portfolios. Using past data and future projections, Monte Carlo simulation is used in portfolio management to forecast a portfolio's possible future performance. Monte Carlo simulation method uses statistical models and historical data to create numerous random samples or scenarios, of future market outcomes. These circumstances are then used to compute the expected risks and returns of a given portfolio under different market conditions.

[18] provides the formula for estimating a portfolio's expected return using Monte Carlo simulation.

$$E(R_p) = \sum_i w_i * E(R_i)$$

$E(R_p)$ expected return of the portfolio, w_i weight of asset i in the portfolio, and $E(R_i)$ expected return of asset i .

Monte Carlo simulation to determine a portfolio's risk is given below:

$$\sigma_p = \sqrt{\sum_i \sum_j w_i * w_j * cov(R_i, R_j)}$$

- i. σ_p is the portfolio risk,
- ii. w_i weight of asset i ,
- iii. w_j weight of the asset j ,
- iv. $cov(R_i, R_j)$ covariance between assets i and j ,

Mean Variance Optimization (MVO)

The mean variance (MV) model was proposed by [1], who won the 1990 Nobel Prize in Economics. For portfolio optimization, mean variance optimization is a popular technique. It includes defining which set of portfolio weights maximizes portfolio return for a given level of risk or minimizes portfolio risk for a given level of expected return. Mean Variance Optimization technique is a quantitative method to portfolio optimization that seeks to find the portfolio allocation that maximizes the expected return for a given level of risk, measured by the portfolio variance or standard deviation [14]

As per [19], MVO problem can be expressed as:

$$\text{minimize } w^T \Sigma w - \gamma \mu^T w$$

$$\text{subject to } e^T w = 1$$

$$w \geq 0$$

w is the vector of portfolio weights, Σ is the covariance matrix of asset returns, γ is a scalar that represents the investor's risk aversion or preference for return, μ is the vector of expected returns of the assets, e is a vector of ones, used to ensure that the weights sum up to 1

$w \geq 0$ means that the weights are non-negative.

The first term $w^T \Sigma w$ represents the portfolio risk, while the second term $\gamma \mu^T w$ represents the expected return.

The objective of Mean Variance Optimization is to minimize the portfolio risk subject to achieving a minimum expected return $\mu^T w$.

The solution to the MVO problem can be found using quadratic programming, a mathematical optimization technique that solves a quadratic objective function subject to linear constraints [20]. This solution finds the best mix of

investments to balance risk and return.

The optimal portfolio weights are given by $w^* = \left(\frac{1}{\gamma}\right) \Sigma^{-1} \mu$, Σ^{-1} is the inverse of the covariance matrix. The tangency portfolio, also known as the efficient frontier portfolio, is the ideal portfolio that results from this process. It is the portfolio with the highest Sharpe ratio, which indicates the excess return per unit of risk.

Sharpe Ratio Optimization

A common risk-adjusted performance metric in portfolio management is the Sharpe Ratio. It calculates a portfolio's excess return over the risk-free rate for each unit of volatility, or portfolio risk. Investors find a portfolio with a greater Sharpe Ratio more appealing since it offers higher returns for a given degree of risk [14].

The Sharpe Ratio can be calculated as follows:

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

R_p is the expected portfolio return, R_f is the risk-free rate and σ_p is the standard deviation of the portfolio return.

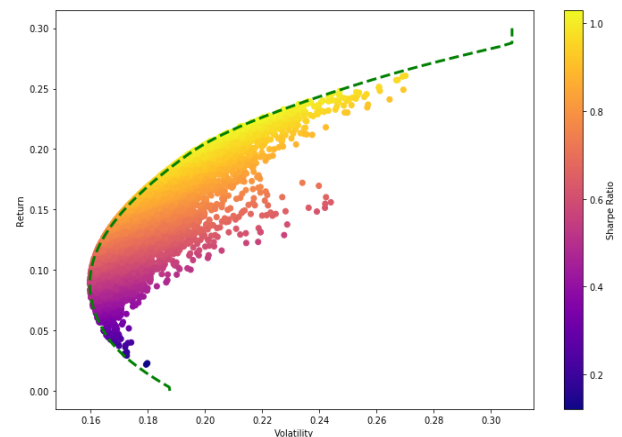
Machine Learning Techniques

Machine learning (ML) algorithms use different mathematical strategies to calculate similarity of assets and establish their boundaries. Clustering is an unsupervised machine learning technique that groups assets based on similarity without requiring labelled data. By applying clustering techniques assets showing similar movement are put in the same cluster. Also, it reduces risk concentration by avoiding to assign too much weights on highly correlated assets.

Workflow of optimization and machine learning model is first machine learning model predict the expected returns and then from ML predicted distribution Monte Carlo simulation generates the random scenarios by sampling.

INTERPRETATION

The Mean-Variance Optimization method, which pursues to maximize the expected return of a portfolio while minimizing its risk [1].



Sources: Created by author.

Figure 2. Efficient Frontier for risk and return

In figure 2, representing the scatter plot of the portfolios produced by Monte Carlo simulations where y axis represents the return and x axis represents the risk.

The "bullet-shaped" curvature confirms a positive correlation structure among the underlying assets, as a perfectly linear boundary would only exist if assets were perfectly correlated. The efficient frontier is represented by green dashed line which identifies the set of optimal portfolios. The result shows that every portfolio located on the upper part of the frontier provides the maximum likely predictable return for a given level of risk. In other words, no other portfolio can deliver better returns without increasing risk. The frontier takes the shape of a hyperbolic curve, while portfolios lying above the curve are not practically achievable.

The analysis further indicates that for a target return of 20%, the minimum achievable volatility is approximately 18%. Similarly, the minimum variance portfolio offers nearly 10% return with around 15% volatility. The simulated portfolio points plotted around the frontier also confirms that the efficient frontier forms the limit of the best probable portfolio combinations. Portfolios positioned below or to the right of the frontier are considered inefficient because they involve higher risk with comparatively lower returns.

The results provide compelling evidence in favor of Harry Markowitz's Modern Portfolio Theory, which holds that investors can better balance risk and return by diversifying their holdings. However, in practical investment situations, factors such as transaction costs, taxes, and possible errors in estimating expected returns and covariance values should also be considered, as these factors can influence the actual performance of optimized portfolios.

CONCLUSION

This study examines the empirical effectiveness of Mean-Variance Optimization (MVO) using Monte Carlo simulation techniques on a technology-focused equity portfolio, [18]. The results support the underlying theory of [1], as the optimal composition of the MVO portfolio requires systematic aggregation of the expected returns, variances and covariances, as opposed to a random, heuristic portfolio construction.

The optimal portfolio (the solution to the optimisation program) maximises the Sharpe Ratio, the well-known risk-adjusted measure of performance devised by [21]. Meanwhile, a naive uniform allocation (40% Apple Inc., 44% Cisco Systems, 2% IBM, 13% Amazon) had an expected volatility of 24.2% and a negative Sharpe Ratio of -0.112. This demonstrates that when assets are held in non-optimised allocations, diversification adds no value but only noise. This exemplifies important implications of modern portfolio theory: diversification pays off only if we hold assets in exactly the right proportions.

The set of portfolios constructed from minimum volatility over different return targets defines upper bounds of opportunity set. Higher-risk portfolios are rewarded in terms

of risk-adjusted return, and portfolios on the Efficient Frontier outperform any portfolio not on the frontier along the connection between risk and return. We can create a detailed picture of the opportunity set using a simple Monte Carlo simulation. This clearly reveals the convex risk-return relationship and demonstrates the manifestation of the supposed convex corner solution (as the upper envelope of the opportunity set).

Nevertheless, the empirical results also show structural limits. The limited asset universe (technology stocks) has a relatively high positive correlation, between Apple and Amazon, which diminishes the efficiency of diversification. High correlations compress the efficient frontier and decrease the possible improvement in return for a given increase in volatility; i.e. the efficiency frontier becomes less convex and shifts outward. This confirms one of the main conclusions of portfolio analysis: that the benefit from diversification strongly hinges on the correlation pattern between stocks. If the portfolio becomes homogenous, contingent or sector-specialised, the set of efficient solutions shrinks [22].

Although the results support the internal logic and computational feasibility of MVO, they also reflect the prevailing fears about estimation error and model instability [18]. As [2] point out, optimal weights may be extremely vulnerable to errors made in inputs (expected returns and covariance matrices). Newer studies, such as [5] work on decision-focused learning models that mash predictive machine learning methods on assets with optimisation models is promising in that respect, which could address instability with optimisation-based dynamic asset allocation.

A fundamental aspect of MPT is that it does not evaluate assets individually but instead focuses on how their returns covary. The study's results expose a structural reality that perfectly aligns with this theory: because the selected technology stocks (particularly Apple and Amazon) exhibited relatively high positive correlations, the efficiency of the diversification was diminished. This high correlation compressed the Efficient Frontier, making it less convex, which verifies the theoretical conclusion that the true benefits of diversification are highly dependent on the correlation patterns between assets. If a portfolio is too homogenous or sector-specialized, the opportunity for efficient optimization shrinks.

Overall, this paper has empirically confirmed mean-variance optimisation as an academically rigorous and analytically valid method of constructing portfolios. The optimised portfolio outperforms its random counterparts in terms of risk adjusted return, and the Efficient Frontier remains a compelling decision-making benchmark. In practice, however, several implementation issues should be considered estimation risk, transaction costs, the non-stationary correlation matrix, and market frictions. Further research could be directed towards robust optimisation procedures, Bayesian shrinkage methods, factor model optimization and machine learning-based forecast distribution to improve out-of-sample performance and

resilience.

Ultimately, the study concludes that building a portfolio by systematically aggregating expected returns, variances, and covariances as dictated by Markowitz's Mean-Variance Optimization consistently outperforms random portfolio construction, thereby empirically confirming the internal logic and validity of these foundational financial theories.

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